Chap 6

Part (A): Variance tests

Same Gaussian models as

Chap 2 : one-sample
Chap 5 : two-samples.

$\Big\}$ focus on σ^2

One-sample: $Y_1, \dots, Y_n \overset{iid}{\sim} N(\mu, \sigma^2)$

$H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 \neq \sigma_0^2$

Same simulation as Chap 2, but different stat:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

Recall from Chap 3 that

$$V \equiv \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

Easier to use V with closed form calculations.

<u>Confidence Interval</u> for σ^2.

- <u>sim:</u> switch σ^2 & s^2 & calc quantiles.

- <u>or:</u>

$$
\begin{aligned}
1-\alpha &= \mathbb{P}\left(q_{\alpha/2} \le V \le q_{1-\alpha/2}\right) \\
&= \mathbb{P}\left(q_{\alpha/2} \le \frac{(n-1)s^2}{\sigma^2} \le q_{1-\alpha/2}\right) \\
&= \mathbb{P}\left(\frac{s^2(n-1)}{q_{1-\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{q_{\alpha/2}}\right)
\end{aligned}
$$

switched

Two-samples

Chap 5: $Y_1, \ldots, Y_{n_y} \overset{iid}{\sim} N(\mu_y, \sigma_y^2)$

& $X_1, \ldots, X_{n_x} \overset{iid}{\sim} N(\mu_x, \sigma_y^2)$

$H_0: \sigma_y^2 = \sigma_x^2 \quad \equiv \quad \sigma_y^2/\sigma_x^2 = 1$

$H_1: \sigma_x^2 \neq \sigma_x^2$

est σ_y^2 w/ s_y^2 & σ_x^2 w/ s_x^2

ratio: $f \equiv s_y^2/s_x^2$ gives a <u>stat.</u>

Chap 3

$$s_y^2 \sim \frac{\chi^2_{n_y-1}\,\sigma_y^2}{n_y - 1}$$

H_0 & $\sigma_y^2 = \sigma_x^2$

$$s_x^2 \sim \frac{\chi^2_{n_x-1}\,\sigma_x^2}{n_x - 1}$$

so
$$\frac{s_y^2}{s_x^2} \sim \frac{\chi^2_{n_y-1}/(n_y-1)}{\chi^2_{n_x-1}/(n_x-1)} \equiv F \sim \underbrace{F_{n_y-1, n_x-1}}_{\text{DoF}}$$

shown

Chapter 6

(B) ANOVA: Analysis of ~~Variance~~ Means

Extension of Chap 5 to 3+ (m) samples

Model: $Y_{ij} \overset{iid}{\sim} N(\mu_j, \sigma^2)$

pooled

$i = 1, \ldots, n_j$
$j = 1, \ldots, m$

$H_0: \mu_1 = \cdots = \mu_m$
$H_1: \mu_j \neq \mu_k \quad j \neq k$

Under H_0:

$$\bar{y} = \frac{1}{n} \sum_{j=1}^{m} \sum_{i=1}^{n_j} y_{ij} \equiv \hat{\mu}$$

$$\sigma^2 = \frac{1}{n-1} \sum_{j=1}^{m} \sum_{i=1}^{n_j} (y_{ij} - \bar{y})^2$$

Unrestricted (under H_1)

Need seperate estimates of parameters

$$\hat{\mu}_j \equiv \bar{y}_{\cdot j} = \frac{1}{n_j} \sum_{i=1}^{n_j} y_{ij} \qquad j = 1, \dots, m$$

$$s_j^2 = \frac{1}{n_j - 1} \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_{\cdot j})^2$$

$$s_{pool}^2 = \frac{\sum_j (n_j - 1) s_j^2}{n - m} \leftarrow m \equiv \# \text{ of } \bar{y}_{\cdot j}$$

Developing a good statistic for testing involves relating H_0 & H_1 mean-based quantities via their residual sum of squares (SS).

Hw: $$\sum_{j=1}^{m} \sum_{i=1}^{n_j} (y_{ij} - \bar{y})^2$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{n_j} (\bar{y}_{\cdot j} - \bar{y})^2 \leftarrow$$

$$+ \sum_{j=1}^{m} \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_{\cdot j})^2$$

SST $\rightarrow$

SSR $\rightarrow$

SSE $\rightarrow$

SST = SSR + SSE $\Leftarrow$

Any part over the whole is a **proportion.**

$$0 < r^2 \equiv \frac{SSR}{SST} = 1 - \frac{SSE}{SST} < 1$$

boiled all my stats into 1 #.

grand stat.

$$f \equiv \frac{SSR/(m-1)}{SSE/(n-m)} = \frac{MSR}{MSE}$$

under H_0 $F \sim F_{m-1,\, n-m}$ b/c numer & denom are each χ^2s