Chapter 1
Coin Flips.

Data: n coin flips $y_1, \ldots, y_n$

$$y_i = \begin{cases} 0 & \text{"tails"} \\ 1 & \text{"heads"} \end{cases}$$

Let θ denote the probability of heads.

$$p(y_i = 1) = \theta \qquad \theta \in [0,1]$$
$$p(y_i = 0) = 1-\theta$$

Given these data, can we determine if the coin is fair? ($\theta = 1/2$?)

3 ingredients

1) Model for the data
2) Hypotheses
3) Statistic

1) <u>Model</u> $Y_i \overset{iid}{\sim} \text{Bern}(\theta) \quad i = 1, \ldots, n$

mass $f(y) = \theta^{y}(1-\theta)^{1-y} \equiv p(Y;\theta)$

independence $p(Y_i, Y_j) = p(Y_i) \cdot p(Y_j)$

so $p(Y_1, \ldots Y_n) = p(Y_1) \cdot \cdots \cdot p(Y_n)$

$$f(y_1, \ldots, y_n) = \prod_{i=1}^{n} f(y_i)$$

$$= \prod_{i=1}^{n} \theta^{y_i}(1-\theta)^{1-y_i}$$

$$= \theta^{\sum y_i}(1-\theta)^{n-\sum y_i}$$

2) <u>Hypotheses</u>

$H_0: \theta = 1/2$

$H_1: \theta \neq 1/2$

3) <u>Statistic</u> (any function of the data values)

How is the data involved in the

1) model using the parameter in the

2) hypotheses.

$$f(y_1,\ldots,y_n) = \theta^{\sum y_i}(1-\theta)^{n-\sum y_i}$$

Ho   a fcn of data

A good stat is $s(y_1,\ldots,y_n) = \sum_{i=1}^{n} y_i$

How to conduct a hypothesis test?

repeat

- Assume Ho is true. 2)
- Generate "data" from the model 1) using Ho
- Calculate the statistic with those "data" 3)

- compare to the observed stat.

A <u>p-value</u> is one way to make a

quantitative comparison.

prob of observing a "more extreme" value of the stat than the one you observed in the data.

if your p-value is < 0.05

→ Reject H_0

otherwise

→ fail to Reject H_0