Q1 plot yield

a. (2 pts) Which test are you using?

median test

b. (3 pts) State the null and alternative hypotheses.

H_0: All methods have the same median yield per acre

H_1: At least two of the methods differ with respect to the median yield per acre

c. (5 pts) Compute the test statistic.

>89	6	3	7	0	$a=16$
≤ 89	3	7	0	8	$b=18$
	$n_1=9$	$n_2=10$	$n_3=7$	$n_4=8$	$N=34$

$$t = \frac{N^2}{ab}\sum_{i=1}^{c} O_{1i}^2/n_i - \frac{Na}{b}$$

$$= 17.6$$

d. (5 pts) Calculate the p-value or rejection region.

$T \sim \chi^2_{c-1} \quad c=4$

$p = P(T > t) = 0.00053$

e. (5 pts) What do you conclude?

Reject Ho & conclude that at least two of the methods ~~yielded~~ provide different median yield per acre.

Q2 fair die

a. (2 pts) Which test are you using?

Goodness-of-fit test.

b. (3 pts) State the null and alternative hypotheses.

H_0: $P(\text{face is } j) = 1/6$ for $j = 1, \ldots, 6$

H_1: $P(\text{face is } j) \neq 1/6$ for at least one class

c. (5 pts) Compute the test statistic.

The expected number of each face is 100.

$t = \sum O_i^2/E_i - N$

so $t = \dfrac{87^2 + 96^2 + 108^2 + 89^2 + 122^2 + 98^2}{100} - 600$

$t = 8.58$

d. (5 pts) Calculate the p-value or rejection region.

$T \sim \chi^2_{c-1}$ $\quad c = 6$

$p = P(T > t) = 0.1270355$

e. (5 pts) What do you conclude?

fail to reject H_0: the die is fair.

Q3 Basketball

a. (2 pts) Which test are you using?

Cochran's test

b. (3 pts) State the null and alternative hypotheses.

H_0: Each sportsman is equally effective in his ability to predict the outcomes of basketball games

H_1: There is a difference in the predictive abilities among sportsmen

c. (5 pts) Compute the test statistic.

The column totals are $C = (8, 10, 7)$, $N = 25$

the row totals are $R = (3, 3, 1, 2, 0, 3, 3, 2, 1, 1, 3, 3)$

$$t = \frac{c(c-1) \sum_{j=1}^{c} (C_j - N/c)^2}{\sum_{i=1}^{r} R_i (c - R_i)} = 2.8$$

d. (5 pts) Calculate the p-value or rejection region.

$T \sim \chi^2_{c-1}$ for $c = 3$

$P(T > t) = 0.246597$

e. (5 pts) What do you conclude?

Fail to reject H_0 & conclude that their predictive ability is the same

Q4 Flint

a. (2 pts) Which test are you using?

Mann-Whitney / Wilcox test

b. (3 pts) State the null and alternative hypotheses. two-tailed

H_0: The flints from areas A & B are of equal hardness

H_1: The flints are not of equal hardness

$\left(\begin{array}{l} H_0: F(x) = G(x) \ \forall x \\ H_1: F(x) \neq G(x) \ \exists x. \end{array} \text{ where } F \equiv A \quad \& \ G \equiv B \right)$

c. (5 pts) Compute the test statistic.

$t =$ sum of ranks from area A

$= 1+2+3+5 = 11$

or $t' = t - n_x(n_x+1)/2 = 11 - 4(5)/2 = 1$

d. (5 pts) Calculate the p-value or rejection region.

$p = 2\min\left(P(T \geq t-1), P(T \leq t)\right)$

or $2\min\left(P(T' \geq t'-1), P(T' \leq t'-1)\right)$

$= 2 \cdot 0.015873 = 0.031746$

e. (5 pts) What do you conclude?

Reject H_0 & conclude that flint from A & B do not have the same hardness

Q5: Heartbeat

a. (2 pts) Which test are you using?

Squared-ranks test

b. (3 pts) State the null and alternative hypotheses. $X \equiv$ men, $Y \equiv$ women

two-tailed test

H_0: X & Y are identically distributed, except possibly for different means

H_1: $Var(X) \neq Var(Y)$

c. (5 pts) Compute the test statistic.

$$t = \sum_{i=1}^{n_x} [R(U_i)]^2 = 1107.5$$

there is 1 tie out of 17 ← (7%) so we use CLT

$$t_1 = \frac{t - n_x \overline{R^2}}{\left[\frac{n_x}{N(N-1)} \sum_{i=1}^{N} R_i^4 - \frac{n_x n_y}{N-1} \right]^{1/2}} = 1.390575$$

d. (5 pts) Calculate the p-value or rejection region.

[full credit] $p \approx 2 \cdot P(Z < -|t_1|) = 2 \cdot 0.082177 = 0.164353$

or $p = 2\min\{P(T \le t), P(T > t)\} = 0.172939$

[partial credit]

e. (5 pts) What do you conclude?

Not enough evidence to Reject H_0: conclude that M & W heartbeats have the same variance

6. MBA v GMAT

a. (2 pts) Which test are you using?

Spearman ρ test

b. (3 pts) State the null and alternative hypotheses.

H_0: GPAs are independent of GMAT scores

H_1: High GPAs tend to be associated with high GMAT scores

(upper-tailed test for positive correlation.

c. (5 pts) Compute the test statistic.

$$\rho = \frac{\sum_{i=1}^{n} R(X_i)R(Y_i) - n\left(\frac{n+1}{2}\right)^2}{\left(\sum_{i=1}^{n} R(X_i)^2 - n\left(\frac{n+1}{2}\right)^2\right)^{1/2}\left(\sum_{i=1}^{n} R(Y_i)^2 \; n\left(\frac{n+1}{2}\right)^2\right)^{1/2}}$$

$$= \frac{589.75 - 2(13/2)^2}{\left(647.5 - 12\cdot(13/2)^2\right)^{1/2}\left(647 - 12\cdot(13/2)^2\right)^{1/2}} = 0.5979$$

d. (5 pts) Calculate the p-value or rejection region.

Since there are ties, we use

$$q = P\left(Z \geq \rho\sqrt{n-1}\right) = P\left(Z \geq 0.5979 \times \sqrt{11}\right)$$
$$= P(Z \geq 1.9568)$$
$$= 1 - P(Z \leq 1.9568)$$
$$= 1 - 0.947815$$
$$= 0.02518$$

e. (5 pts) What do you conclude?

Reject H_0 & conclude that high GPAs are associated with high GMAT scores

Q7 Chemical detection kits.

a. (2 pts) Which test are you using?

~~Kruskal~~ Kruskal-Wallis

b. (3 pts) State the null and alternative hypotheses.

H_0: All four ~~[illegible]~~ distributions for kits are identical

H_1: At least one of the kits' distributions yield larger observations than at least one of the other kits.

c. (5 pts) Compute the test statistic.

~~[illegible]~~

$$T = \frac{1}{S^2}\left(\sum_{i=1}^{4} \frac{R_i^2}{n_i} - \frac{N(N+1)^2}{4}\right)$$

$$S^2 = \frac{1}{N-1}\left(\sum_{ij} R_{ij}^2 - \frac{N(N+1)^2}{4}\right) = 136.6538$$

$$T = 8.215283$$

d. (5 pts) Calculate the p-value or rejection region.

$$T \sim \chi^2_{4-1} \qquad \varphi = P(T > t) = 1 - P(T < 8.21)$$

$$= 1 - 0.95823$$

$$= 0.04176$$

e. (5 pts) What do you conclude?

reject H_0 & conclude that there is a difference in the distributions for the kits.

Q 8 Contingency Coefficients

observed

H \ W	Clinton	Trump	Other	
Clinton	12 (16)	22 (20)	6 (4)	40
Trump	25 (20)	21 (25)	4 (5)	50
Other	3 (4)	7 (5)	0 (1)	10
	40	50	10	100

expected in parens $\left(E_{ij} = \frac{n_i c_j}{N} \right)$

$$T = \sum_{ij} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 6.34$$

(A)

Cramer's $R_1 = \frac{T}{N(\min\{r,c\} - 1)} = \frac{6.34}{100 \cdot (3-1)} = 0.0317$

(pretty low)

$C_{cc} = \sqrt{R_1}$

$= 0.178$ ranges between 0 & 1

$$R_2 = \sqrt{\frac{T}{N+T}} = \sqrt{\frac{6.34}{100+6.34}} = 0.244$$

$q = \min\{r,c\}$ ranges from $\left[0, \sqrt{\frac{q-1}{q}}\right] = 0.816$

$$R_3 = \frac{T}{N} = 0.0634$$

with a range of $[0, q-1] = 2$

all three are low relative to their maximum values.

(B) for the 2x2 Table

$a = 12$	$b = 22$	$r_1 = 34$
$c = 25$	$d = 21$	$r_2 = 46$
$c_1 = 37$	$c_2 = 43$	$N = 80$

$$R_\phi = \left(\frac{ad - bc}{\sqrt{r_1 r_2 c_1 c_2}}\right) = -0.1889$$

A low negative association

(C) No change.