1. Metro crime

a. (3 pts) State the null and alternative hypotheses.

$H_0: p = 0.7$ where p is the proportion
$H_1: p \neq 0.7$ of passengers that have not
whitnessed a crime

two-tailed test

b. (5 pts) Compute the test statistic.

a binomial test

$t = 10$ = # of passengers not whitnessing a crime

$T \sim Bin(n = 12, p^* = 0.7)$

c. (7 pts) Calculate the p-value.

$$p \cong 2 \cdot \min\left(P(T \leq 10), P(T \geq 10)\right)$$
$$= 2 \cdot \min\left(P(T \leq 10), 1 - \overset{0.74718}{P(T \leq 9)}\right)$$
$$= 2 \cdot \min(0.915, 0.283) = 2 \cdot 0.283 \cong 0.51$$

d. (5 pts) What do you conclude?

fail to reject H_0, conclude
$p = 0.7$. passengers between 11pm & 3am
have not whitnessed a crime on a
Metro train.

2. geyser

a. (3 pts) State the null and alternative hypotheses.

$H_0: P(X \le 60) \ge 0.5$

$H_1: P(X \le 60) < 0.5$, where X is the time interval between eruptions.

(~~right~~-tailed test) ~~upper~~ lower ~~quantile test~~

b. (5 pts) Compute the test statistic.

a quantile test

$t_1 = 38$ & $T_1 \sim Bin(n = 112, p = 0.5)$

c. (7 pts) Calculate the p-value.

in this ~~upper~~ lower-tailed test $p = P(T_1 \le t_1)$

$p = P(T_1 \le 38)$ $T \sim Bin$

$p = 0.000130$

d. (5 pts) What do you conclude?

At the 5% level, reject H_0 — the median time interval between eruptions is greater than 60 minutes

3. Manufactur

a. (3 pts) State the null and alternative hypotheses.

If we write as "+" the event that customers prefer B to A & "−" if A is preferred to B, then $H_0: P(+) \leq P(-)$

$H_1: P(+) > P(-)$ an upper-tailed test

b. (5 pts) Compute the test statistic.

This is a sign test $t = 8$ & $n = 9$

where $T \sim Bin(n=9, p=1/2)$

c. (7 pts) Calculate the p-value.

$$p = P(T \geq t) = P(T \geq 8)$$
$$= 1 - P(T \leq 7)$$
$$= 1 - 0.98469$$
$$= 0.0195$$

d. (5 pts) What do you conclude?

At the 5% level we reject H_0, and conclude that the consumer population prefers B to A

4. Debate pres

a. (3 pts) State the null and alternative hypotheses.

Let $0 \equiv$ Democrat, $1 \equiv$ Republican

$X_i \equiv$ preference before debate, $Y_i \equiv$ preference after.

$H_0 : P(X_i = 0, Y_i = 1) = P(X_i = 1, Y_i = 0) \equiv$ alignment not altered

V. $H_1 : P(X_i = 0, Y_i = 1) \neq P(X_i = 1, Y_i = 0) \equiv$ alignment altered

two-tailed test

b. (5 pts) Compute the test statistic.

McNemar test

X \ Y	D	R	total
D	a=63	b=21	84
R	c=4	d=12	16
			100

$n = b + c = 25 > 20$

so $t_1 = \dfrac{(b-c)^2}{b+c} = 11.56$

$T_1 \sim \chi^2_1$

c. (7 pts) Calculate the p-value.

$$\begin{aligned} \psi = P(T_1 > t_1) &= P(T_1 > 11.56) \\ &= 1 - P(T_1 \leq 11.56) \\ &= 1 - 9.999326 \\ &= 0.000673 \end{aligned}$$

d. (5 pts) What do you conclude?

Reject H_0 @ 5% level and conclude voter alignment has been altered.

5. Precip

a. (3 pts) State the null and alternative hypotheses.

pairing off: (45.25, 41.05), (45.83, 33.72), (41.77, 45.73)
(36.26, 37.90), (45.37, 41.72), (52.25, 36.07), (35.37, 49.83)
(56.16, 36.24), (35.37, 39.90). 58.32 omitted since n' odd (19)

test H_0: $P(+) = P(-)$ v. $H_1: P(+) \neq P(-)$

two-tailed test.

b. (5 pts) Compute the test statistic.

Cox & Stuart test for trend

$n = 9$ $t = \#$ of "+" 4 $T \sim Bin(n=9, p=1/2)$

c. (7 pts) Calculate the p-value.

$p \approx 2 \cdot \min\{P(T \leq t), P(T \geq t)\}$
$= 2 \cdot \min\{P(T \leq 4), 1 - P(T \leq 3)\}$
$= 2 \cdot \min(0.5, 1 - 0.25) = 2 \cdot \min(0.5, 0.75)$
$= 2 \cdot 0.5 = 1$

d. (5 pts) What do you conclude?

Not enough evidence to reject H_0;
conclude that no trend exists

6. Defective

a. (3 pts) State the null and alternative hypotheses.

Let p_1 be the proportion defective in load 1 & p_2 in load 2.

Then, test $H_0: p_1 \neq p_2$

v. $H_1: p_1 = p_2$

a two-tailed test

b. (5 pts) Compute the test statistic.

Proportion test

	def	non-def	tot
load 1	$O_{11} = 13$	$O_{12} = 73$	86
2	$O_{21} = 17$	$O_{22} = 57$	74
tot	30	130	160

$$t_1 = \frac{\sqrt{N}\,(O_{11}O_{22} - O_{12}O_{21})}{\sqrt{n_1 n_2 C_1 C_2}} = -1.2695 \quad T_1 \sim N(0,1)$$

c. (7 pts) Calculate the p-value.

two-tailed test $p = 2 \cdot P(T_1 \geq |t_1|)$

$= 2 \cdot P(T_1 \geq 1.2695)$

$= 2 \cdot (1 - P(T_1 \leq 1.2695))$

$= 2 \cdot (1 - 0.897868) = 0.2042$

d. (5 pts) What do you conclude?

Accept H_0 @ 5% level ≡ the two carloads are about the same.

7. Chipmunks

a. (3 pts) State the null and alternative hypotheses.

let p_1 denote chipmunks ~~close~~ trilling probability close to home & p_1 far from home.

$H_0: p_1 \le p_2$

$H_A: p_1 > p_2$

an upper-tailed test

b. (5 pts) Compute the test statistic.

fisher's exact test

	trill	not	
close	$x=16$	$r-x=8$	$r=24$
far	$c-x=3$	$N-r-c+x=18$	$N-r=21$
	$c=19$	$N-c=26$	$N=45$

$t_2 = x = 16$

$T_2 \sim \text{HyperGeom}(\cancel{}\ N=45, r=24, c=19)$

c. (7 pts) Calculate the p-value.

$$\varphi = P(T_2 \geq t_2) = P(T_2 \geq 16) = 1 - P(T_2 \le 15)$$

$$\approx 1 - 0.9995679$$

$$= 0.000432$$

d. (5 pts) What do you conclude?

Reject H_0 @ 5% level and conclude that chipmunks do trill more when closer to home.

8. Weight CI

$\alpha = 0.1$

r s.t $P(T \le r) = \alpha/2$ $\quad T \sim \mathrm{Bin}(n=20, p=1/2)$

s s.t $P(T \le r) = 1-\alpha/2$ $\quad T \sim$ ''

$r = 6 \quad s = 14 \quad$ CI: $[103, 137]$

$\alpha_1 = P(T \le r-1) = 0.0206947$

$\alpha_2 = 1 - P(T \le s-1) = 1 - 0.9423409$

$= 0.0576915$

$1 - \alpha = 1 - (\alpha_1 + \alpha_2) = 0.92164$

9. Steel Rods

tol limit

$r=6$, $m=6$

$X_{1-\alpha}$ where $T \sim \chi^2_{2(r+m)} \equiv \chi^2_{24}$

the 99th quantile is 42.97982

$$\frac{1}{4} \cdot X_{1-\alpha} \frac{(1+q)}{(1-q)} + \left(\frac{1}{2}\right)(r+m-1)$$

$= 209.6541$

$\Rightarrow n = 210.$

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10. Fat

$r=2$, $m=2$

$X_{1-\alpha}$ where $T \sim \chi^2_{2\cdot(r+m)} = \chi^2_8$

$$q = \frac{4n+2(r+m-1) - X_{1-\alpha}}{4n+2(r+m-1) - X_{1-\alpha}} = 0.923$$

92%